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Optimal Location of Actuators for Active Damping of Vibration

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A computationally efficient procedure for selection of placement of actuators is described in this paper. This method is formulated in modal coordinates and is based on the so-called progressive collapse analogy of an adjoint structure made of a hypothetical elastic-brittle material and having the same shape as the eigenmodes of the structure to be controlled. The computer simulation of progressive collapse for the adjoint structure provides us (as a side effect) with the location of the most sensitive spots, the best for placement of actuators. The presented methodology can be applied to damping of free vibration as well as to quasistatic shape control problems.

Introduction

LARGE space structures with low weight requirements will be very flexible, and therefore they will need some type of active control system to suppress vibration and to maintain shape specifications.[1]

It is known that the effectiveness of a structural control system is strongly dependent on actuator and sensor locations. Controllability and observability of a structure control system, which depend primarily on actuator and sensor location, will have a major influence on the efficiency of the control system and the control effort required to satisfy design requirements.

The problem of actuator/sensor location in dynamic systems has not been extensively treated despite its importance. Actuator locations are often selected before the control system is designed, or they are specified based on other considerations. In Ref. 2, attention is focused on sensor placement and sensor system evaluation. The optimal and suboptimal sensor locations are designed with location constraints for a given plant matrix. In Ref. 3, minimum control energy is the criterion for calculating the optimal actuator locations. Using a different heuristic approach, the case of quasistatic loads is treated in Ref. 4, and a statistical approach is applied to search for the optimal location of actuators in Ref. 5. Recently, a new approach for designing optimal locations for actuator/sensor devices as well as sizing of structural members has been presented.[6]

The aim of this paper is to present a new and efficient method for determining optimal locations for actuators designed on the basis of a progressive structural collapse analogy (PCA). It can be applied to designing large structures, where other methods based on general optimization procedures are not efficient enough. The PCA method simulates a behavior of an adjoint structure made of elastic-brittle material, shaped as one of the structural eigenmodes and loaded by compression of two rigid stamps with a progressively decreasing gap in between them. The side effect of this progressive collapse analysis is the location of brittle refractions, with this location being the objective of the search. It will be shown in the paper that the distribution of refractions determined by the PCA method will coincide with the optimal distribution of actuators. The proposed approach can be applied to active damping problems as well as to quasistatic shape control (cf. Ref. 4).

It is important to note that the proposed model of structural control contains the structure itself as well as the controlling devices (actuators) modeled by so-called "active distortions." The theory and applications of the corresponding virtual distortion method (VDM) based on a modified constitutive relation between local stresses, strains, and virtual ("active" in the application discussed here) distortions were presented recently.[7-9] The standard approach, considering control forces as external loads (cf. Ref. 6) can lead to another (not adequate) placement for actuators. Moreover, applying, for example, the piezoelectric or thermal actuators it is necessary to include the coupled fields interactions between electromagnetic, thermal fields and elastic deformations (cf. Ref. 10). The VDM method is open to all of these interactions (through the field of virtual distortions). One can also add that the same VDM concept used for simulation of active distortions generated by actuators is applied for numerical efficient simulation of structural modifications in the PCA design process.

In this paper structural analysis is constrained to small deformations.

Concept of VDM Simulation of Structural Remodeling and Control

Let us consider a body occupying a domain Ω in R^3 limited by the boundary Γ and supported on some part of Γ_u of it. The solid is loaded by body forces f_i and boundary tractions p_i on its free boundary Γ_p (cf. Fig. 1a). On the other hand, let us consider the same body (with the same support conditions) with virtual distortions ϵ_{ij}^0 generated inside the domain Ω (Fig. 1b). For linear elasticity the set of equations describing the resulting strains and stresses for each case can be expressed as follows (cf. Ref. 7):

$$\mathcal{R}(\sigma_{ij}^L, f_i, p_i) = 0, \qquad \mathcal{C}(\epsilon_{ij}^L) = 0, \qquad \sigma_{ij}^L = E_{ijkl}\,\epsilon_{kl}^L \tag{1a}$$

$$\mathcal{R}(\sigma_{ij}^R) = 0, \qquad \mathcal{C}(\epsilon_{ij}^R) = 0, \qquad \sigma_{ij}^R = E_{ijkl}\,(\epsilon_{kl}^R - \epsilon_{kl}^0) \tag{1b}$$

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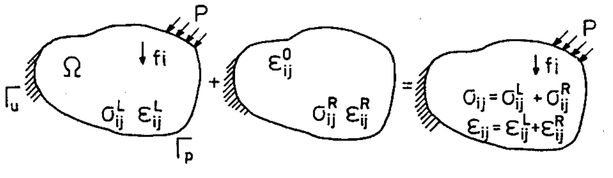

Fig. 1 Elastic body: a) loaded body; b) virtual distortion state; and c) final state.

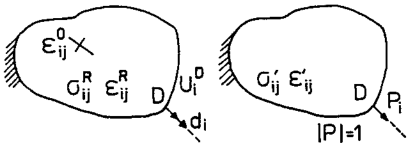

Fig. 2 Control of displacement in point D: a) structure and b) adjoint structure.

where $\mathcal{R}(\cdots) = 0$ denotes the equilibrium conditions (together with the boundary conditions on Γ_p), $\mathcal{C}(\cdots) = 0$ the compatibility conditions (together with the boundary conditions on Γ_u), and E_{ijkl} the elasticity tensor. Superscripts L and R refer to the external load case (Fig. 1a) and to the virtual distortion case (Fig. 1b), respectively. Superposing linear equations (1a) and (1b), the following state of final strains ϵ_{ij} and stresses σ_{ij} can be determined:

$$\mathcal{R}(\sigma_{ij}, f_i, p_i) = 0 \tag{2a}$$

$$\mathcal{C}(\epsilon_{ij}) = 0 \tag{2b}$$

$$\sigma_{ij} = E_{ijkl}(\epsilon_{kl} - \epsilon^0_{kl}) = E'_{ijkl}\epsilon_{kl} \tag{2c}$$

where

$$\sigma_{ij} = \sigma^L_{ij} + \sigma^R_{ij}, \qquad \epsilon_{ij} = \epsilon^L_{ij} + \epsilon^R_{ij} \tag{3a}$$

$$E'_{ijkl} = E'_{ijkl}(E_{ijkl}, \epsilon_{kl}, \epsilon^0_{kl}) \tag{3b}$$

Equation (3b) shows that self-equilibrated stresses σ^R_{ij} and compatible deformations ϵ_{ij} caused by some properly determined virtual distortions ϵ^0_{ij} can simulate modifications of material properties (from E_{ijkl} to E'_{ijkl}) for a deformable body. In particular, the behavior of an elastic-brittle structure can be described by the model of elastic structure with virtual distortions (simulating brittle fractures) generated in overloaded zones.[8]

On the other hand, in problems of active structural control, the "active distortions" ϵ^0_{ij} are no longer fictitious and are generated (in real time) by a set of actuators. Stresses σ^R_{ij} and strains ϵ^R_{ij} [Eqs. (1–3)] then describe the real corrections of resultant stresses and strains due to controlled active distortions. Some applications of the VDM method to simulation of active structural control were previously discussed in cases of quasistatic stress control,[11] quasistatic strain control,[8] active damping control,[9] and local vibration isolation.[12]

The physical model of applied devices is outside of our consideration. However, all of them (based on piezoelectric wafers, shape memory alloys, thermal effects, or pneumatic devices, etc.) can be described as controlled sources of virtual (active) distortions.

Control of Structural Deflection

In this section the case of control whose purpose is the modification of the local deflections of the structure is discussed. A structure where displacement u^D_i at a point D along the directions d_i is to be modified is shown in Fig. 2. In Fig. 2a the structure has virtual distortion ϵ^0_{ij} generated locally in some position to be determined, and in Fig. 2b an adjoint structure with the same support conditions and loaded by unit force $p_i(|\boldsymbol{p}| = 1)$ applied at D along d_i is considered.

From the virtual work principle it follows that

$$p_i u^D_i = |\boldsymbol{p}|\,|\boldsymbol{u}^D| = |\boldsymbol{u}^D| = \int_\Omega \sigma'_{ij}\,\epsilon^R_{ij}\,\mathrm{d}v \tag{4}$$

since p_i is a unit vector and has the same direction as u^D_i. In Eq. (4) σ'_{ij} denotes stresses caused by p_i, ϵ^R_{ij} denotes the strains caused by distortion ϵ^0_{ij} (having a Dirac-function-like distribution concentrated in one determined point of the structure), and u^D_i is the displacement in the point D along d_i due to this distortion. Modifying the right-hand side of Eq. (4), the following expressions are obtained:

$$\int_\Omega \sigma'_{ij}\,\epsilon^R_{ij}\,\mathrm{d}v = \int_\Omega \sigma'_{ij}\,(\epsilon^R_{ij} - \epsilon^0_{ij})\,\mathrm{d}v + \int_\Omega \sigma'_{ij}\,\epsilon^0_{ij}\,\mathrm{d}v$$

$$= \int_\Omega E_{ijkl}\epsilon'_{kl}(\epsilon^R_{ij} - \epsilon^0_{ij})\,\mathrm{d}v + \int_\Omega \sigma'_{ij}\epsilon^0_{ij}\,\mathrm{d}v \tag{5}$$

where the various ϵ'_{ij} are strains caused by p_i. Since the constitutive relation, Eq. (1b), and the fact that E_{ijkl} is a symmetric tensor, σ^R_{ij} is a self-equilibrated state of stresses, and ϵ^L_{ij} is a compatible state of deformations (so both are orthogonal), the right-hand side of Eq. (5) is equal to

$$\int_\Omega \epsilon'_{kl}\,E_{klij}(\epsilon^R_{ij} - \epsilon^0_{ij})\,\mathrm{d}v + \int_\Omega \sigma'_{ij}\epsilon^0_{ij}\,\mathrm{d}v$$

$$= \int_\Omega \epsilon'_{kl}\sigma^R_{kl}\,\mathrm{d}v + \int_\Omega \sigma'_{ij}\epsilon^0_{ij}\,\mathrm{d}v = \int_\Omega \sigma'_{ij}\epsilon^0_{ij}\,\mathrm{d}v \tag{6}$$

Finally, making use of Eqs. (4–6), the following expression determining the value of displacement caused in point D in direction d_i (by distortion generated in the body) can be obtained:

$$u^D = \int \sigma'_{ij}\epsilon^0_{ij}\,\mathrm{d}v \tag{7}$$

It follows from the previous equation that the optimal locations for virtual distortion correcting the displacement in point D are the points where the value of the local product $\sigma'_{ij}\epsilon'^0_{ij}$ (where ϵ'^0_{ij} is unitar) is maximal. Therefore, the location of actuators should be correlated with points of extremal stresses σ'_{ij} in the adjoint structure.

No constraints imposed on virtual distortions were taken into account in the preceding considerations. However, if for technical reasons only some special states of active distortions can be generated, it is necessary to impose the corresponding constraints on distortions.

The subsequent analysis will deal with damping of free structural vibration. Eventual application of a modification of the methodology presented later based on the result of Eq. (7) to problems of dynamic loading will be considered separately.

Optimal Location of Actuators for Flexible Structures

In this section a procedure for selecting the optimal position of actuators, based on the result of Eq. (7), for flexible structures is presented. The procedure is demonstrated on an example of truss-beam structure. However, the discussed concept is more general and can be applied to continuous shells as well, which will be demonstrated in a separate paper.

To be able to damp any free vibration of the flexible structure or to correct any disturbances of the structural shape, it is convenient to apply the modal decomposition of the problem and to control each mode separately.

Let u_i be an eigenmode (a component of structural displacements to be controlled) computed from the vibration eigenvalue problem:

$$(K - \lambda M)u = 0 \tag{8}$$

where K and M are the global stiffness and mass matrices. For shape control a similar formulation can be derived. It is assumed that the aim of this control is to minimize the maximal deflection:

$$\min \max |u_i| \tag{9}$$

To determine the optimal location for actuator devices, a step-by-step methodology based on the concept of PCA of an adjoint structure is developed. In this procedure the progressively decreasing limit values δ_k on maximal local deflections are imposed:

$$|u_i| \leq \delta_k, \qquad (\delta_{k+1} < \delta_k) \tag{10}$$

The adjoint structure is made of a hypothetical elastic-brittle material, formed in the shape of the eigenmode u_i, and is loaded by compression forced by two rigid stamps (cf. Fig. 3). In the first step of the procedure, the constraint of Eq. (10) is satisfied, which follows from the assumption that this structure is under compression of two rigid stamps [both of them are in the same shape of the initial structural configuration since, according to Eq. (9), the purpose of the control is the

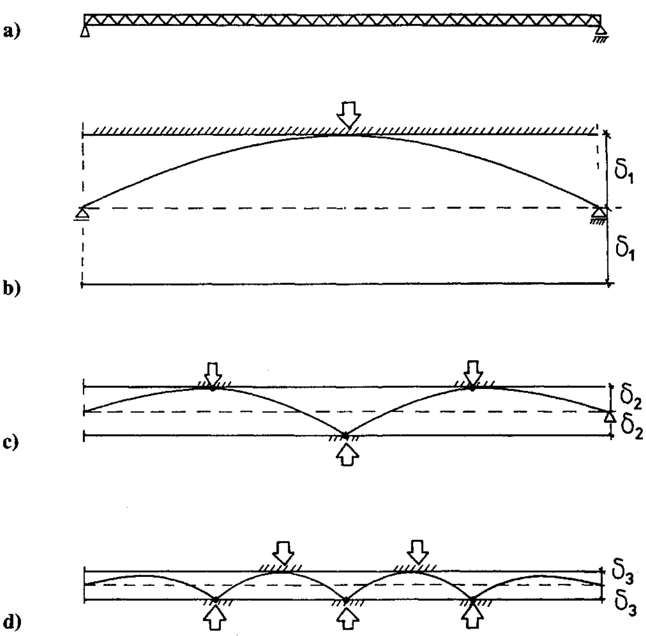

Fig. 3 Progressive collapse analogy: a) flexible truss beam; b) adjoint structure for the first mode; c) first step collapse; and d) second step collapse.

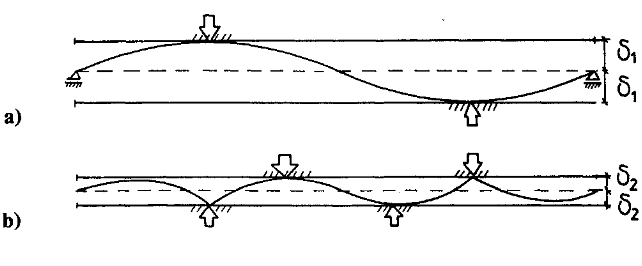

Fig. 4 Progressive collapse analogy: a) adjoint structure for the second mode and b) first step collapse.

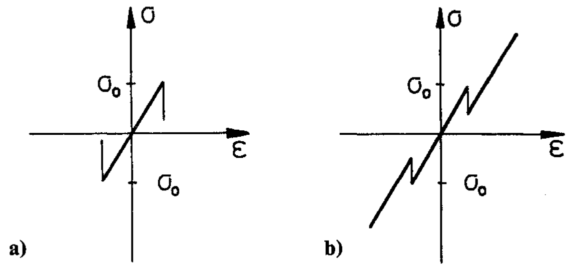

Fig. 5 Elastic-brittle model: a) ideal actuators and b) constrained actuators.

minimization of deflections from this initial configuration] with gap $2\delta_1$ in between them. Equation (7) shows that localization of fracture distortions ϵ_i^0 generated in overloaded elements (ϵ_i^0 simulates fracture causing local stresses to vanish) coincides with the optimal distribution of actuators. Propagation of cracks (under increasing load) leads naturally to the generation of local kinks in the flexible structure. It results in a drastic decrease of the gap value (from $2\delta_1$ to $2\delta_2$) and can be considered as the second step in the PCA process. In this second step a new set of optimal positions for actuators is detected in the points where new fractures are generated. This process goes on up to the moment when enough optimal locations for actuators have been designed. It is important to point out that, to satisfy the criterion of Eq. (9), in each step the initial configuration of the eigenmode has to be in the middle position in between both stamps. Also, an assumption has been made that the local modification of mass and stiffness distribution due to installation of actuator devices is small enough not to affect the solution of the eigenvalue problem of Eq. (8). It is justified applying, for example, piezoelectric devices.

Let us describe this process on an example of the first mode of deformation of the simply supported flexible truss shown in Fig. 3a. The adjoint structure shown in Fig. 3b is made of an elastic-brittle material, has the shape of the first eigenmode, and is loaded by the pressure generated by two rigid stamps compressing the structure. It is apparent from Fig. 3b that in the first step only the middle point is selected as a location for an actuator. The same approach can be applied to the second stage of the progressive collapse of the adjoint structure (Fig. 3c), two new locations being selected, and so on (Fig. 3d). Similarly, the process of the subsequent generation of locations for brittle fractures (locations of actuators) for the second mode is shown in Fig. 4.

The process of structural collapse shown in Figs. 3 and 4 corresponds to the behavior of the structure made of the brittle material shown in Fig. 5a. In Fig. 5a the stress limit σ_0 can be chosen arbitrarily since its value has no influence on the results of the analysis.

If values of locally generated active distortions are limited,

$$|\epsilon_i^0| \leq \bar{\epsilon} \tag{11}$$

where $\bar{\epsilon} > 0$, then the physical model shown in Fig. 5b should be applied. In this case it cannot be assumed that any rotation angle is possible in the fracture kink, and the progressive collapse process would generate different fracture locations for different values of $\bar{\epsilon}$. In Fig. 6 the first step of progressive collapse corresponding to the constitutive equation demonstrated in Fig. 3b is shown.

The sequence of generation of locations for actuators to control each mode has been divided into stages, the number of positions growing with the number of steps. An important question is how to define the modes to be controlled and the number of steps for every mode. Normally, the eigenvalues of the first two or three modes are much lower for flexible structures than the eigenvalues for the higher modes. So, having solved the eigenvalue problem, it can be decided how

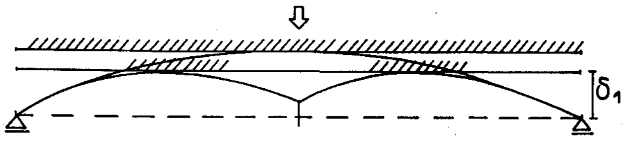

Fig. 6 Progressive collapse with restrictions [Eq. (11)].

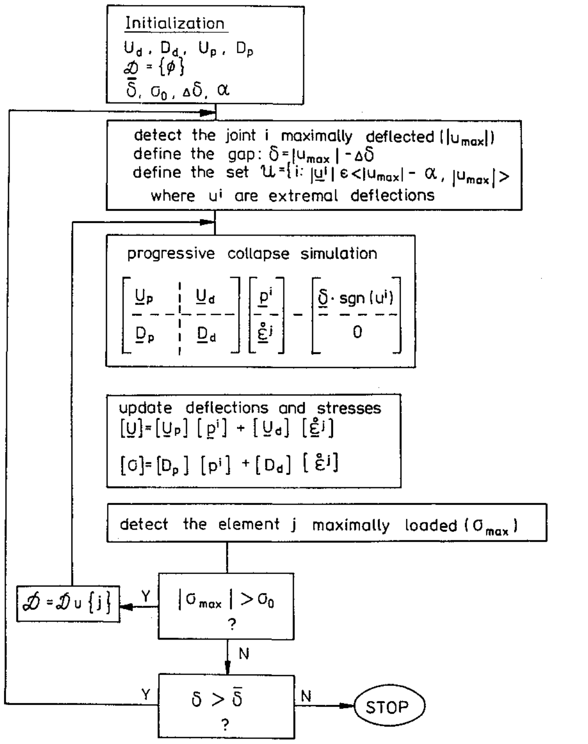

Fig. 7 Algorithm flowchart for simulation of progressive collapse.

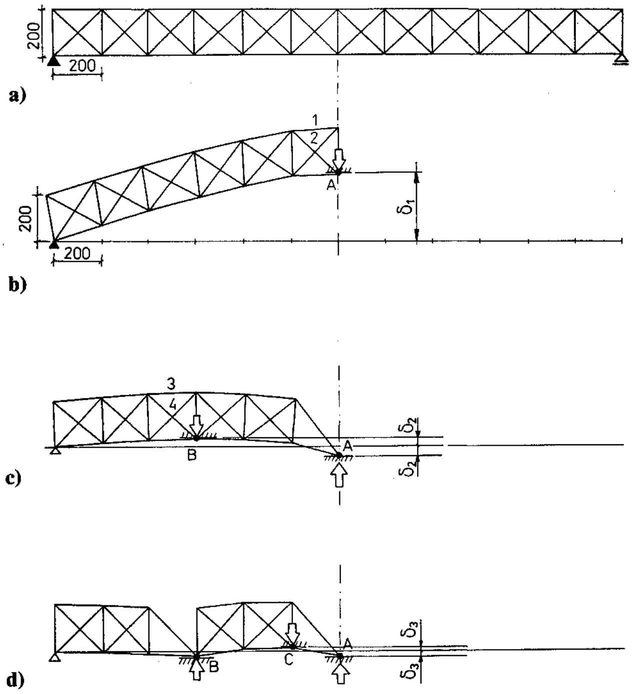

Fig. 8 Sequence of the optimal location for actuators to control the first mode.

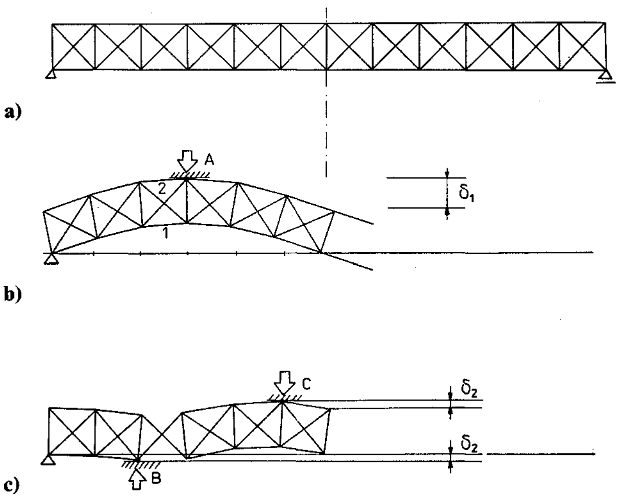

Fig. 9 Optimal location of actuators to control the second mode.

many eigenmodes should be controlled. It is correlated with the eigenvalues of the first modes because the structural deflection for each mode strongly depends on the corresponding eigenvalues. Taking into account n first modes, the weight coefficient for numbers of stages to be included to control each mode (assuming that the total number m of stages to be analyzed is big enough) could be determined by the following formula:

$$\beta_k = \alpha_k \Big/ \sum_{l=1}^{n} \alpha_l \tag{12}$$

where $\lambda_k = 1/\alpha_k$ is the kth eigenvalue, preferring first modes (causing normally much bigger deflections than the higher modes).

Finally, if eigenvalues are determined and the number n is defined, the proportion of the number of stages for each mode is determined by Eq. (12). Then, solving the problem of progressive collapse for each mode and determining numbers of locations due to each stage of each mode, the final number and location of actuators can be determined. Assuming at least one stage of collapse for each mode taken into account, it is possible to decompose the control process for each mode independently. The weight coefficients of Eq. (12) cause the number of stages (and therefore also the number of actuators) for first modes (with small eigenvalues) to be bigger than for higher modes. In the preceding example, extra assumptions that the control action does not disturb the symmetry or skew symmetry of the modes and also that the number of analyzed stages m is not smaller than the number n of modes to be controlled were taken into account. Nevertheless, the analysis of the problem in the case $m < n$ can also be undertaken.

Numerical Procedure Simulating the Process of Progressive Collapse

The numerical procedure simulating the process of progressive collapse in elastic-brittle structures (the adjoint structure shaped as one of the structural modes) under the compression generated by two rigid stamps with gradually decreasing gap 2δ can be described in the following stages.

1) Initialize the following quantities:

- $\mathfrak{D} = \{0\}$ = set of locations of brittle fractures
- $\Delta\delta$ = step of gap decrease
- α = tolerance to define the current set of locations in contact with stamps
- $\bar{\delta}$ = assumed limit for deflection for the adjoint structure
- σ_0 = stress limit

2) Determine the maximal local deflection $|u_i| = u_m$.

3) Update the gap value $\delta = u_m - \Delta\delta$.

4) Define the set of locations that are currently in contact with stamps, $\mathfrak{U} = \{i: |u^i| \in [u_m - \alpha, u_m]\}$ where u^i are locally extremal deflections.

5) Analyze the structure with virtual (brittle) distortions generated in locations defined by the set $\mathfrak{D}$ and loaded by the forced compressing displacements δ generated in locations determined by the set $\mathfrak{U}$.

6) Determine the maximally loaded location j: $|\sigma^j| = \sigma_{\max}$ (several locations could be selected).

7) If $\sigma_{\max} \geq \sigma_0$, then $\mathfrak{D} = \mathfrak{D} \cup \{j\}$ and go to step 5.

8) If $\delta \geq \bar{\delta}$, then go to step 2, or else stop.

The stress limit σ_0 can be defined arbitrarily and does not affect the distribution of the brittle fractures.

Eventual checking for active constraints, Eq. (11), imposed on virtual distortions and corresponding corrections of this state should be included (if necessary) in steps 5–7 of the algorithm.

This process is described more precisely in Fig. 7 where superscripts i and j correspond to the joint and element number, respectively.

The process can be continued up to the moment when the assumed limit for structural deflection $\bar{\delta}$ is reached. The VDM simulation technique applied in the preceding PCA (Fig. 7) is based on the use of the four following influence matrices: $\boldsymbol{U}_p$ and $\boldsymbol{D}_p$ denote displacements and stresses, respectively, caused in the structure by the unit forces applied in the joints "i" (where $i \in \mathfrak{U}$) in the direction determined by the stamp movement; and $\boldsymbol{U}_d$ and $\boldsymbol{D}_d$ denote displacements and stresses, respectively, caused in the structure by the unit virtual distortions applied in the elements "j" (where $j \in \mathfrak{D}$). As mentioned earlier, it is assumed that the local mass and stiffness distributions are not modified by replacing a normal element by an active one. Therefore, adding new elements to sets $\mathfrak{U}$ and $\mathfrak{D}$ means adding new rows and columns to matrices $\boldsymbol{U}_d$ and $\boldsymbol{D}_d$, without modifications of the previously determined parts.

The numerical efficiency of the presented method arises from the fact that the technique of simulation of local corrections applied to the states of stresses and deflections is considerably cheaper than the renewed global analysis (with stiffness matrix reformation). This advantage is especially important for the problems where progressive collapse analysis requires calculation of a long sequence of structural modifications.

Analyzing the results of the development of brittle fractures, the final decision reaching some compromise between

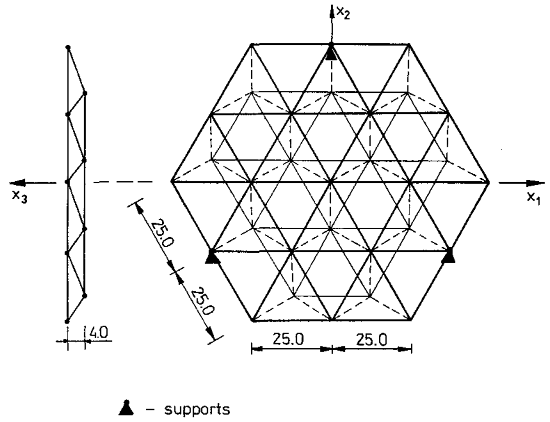

Fig. 10 Antenna backup truss structure.

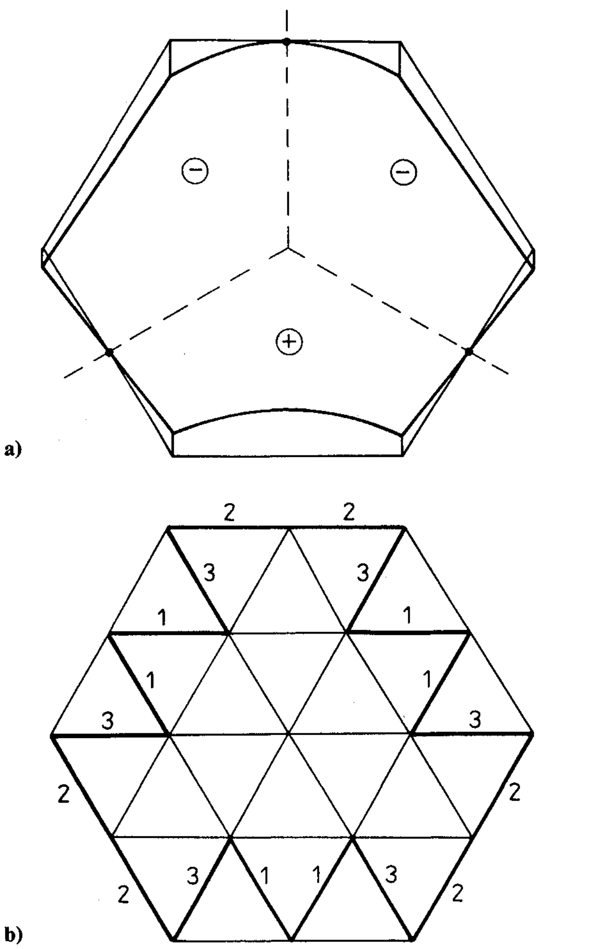

Fig. 11 Location of actuators to control the first mode: a) nodal lines and b) optimal sequence of locations.

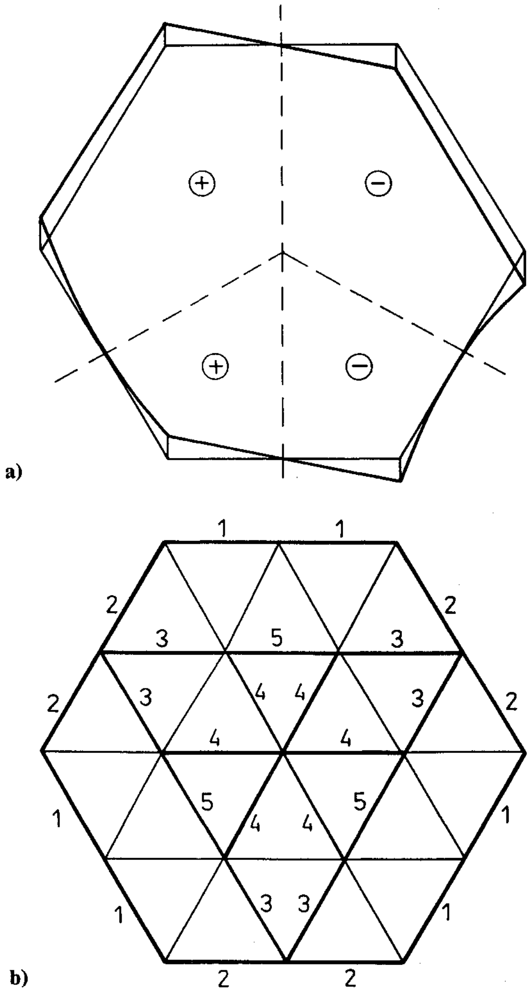

Fig. 12 Location of actuators to control the second mode: a) nodal lines and b) optimal sequence of locations.

the number of actuators and reduction of structural deflection should be made.

Examples

The example of a two-dimensional truss-beam structure shown in Fig. 8a with the uniform area of cross sections $A = 1.0$ cm^2, mass density $\rho = 0.1$ kg/cm^2, and Young's modulus $E = 10^7$ kg/cm^2 for all elements was analyzed.

The adjoint structure shaped as the first eigenmode (Fig. 8b, only one-half of the symmetric structure is shown) was loaded by displacement forced in joint A (location of the maximal deflection). As a result of load increment, the sequence of fractures of two elements (nos. 1 and 2) was generated. Then the kinematic (stressless) modification of structure configuration (Fig. 8c) with the significant decrease of the gap from the value δ_1 to $\delta_2 = 0.11\delta_1$ was caused. This stage of structural collapse corresponds to replacing of elements 1 and 2 by active elements generating active distortions, causing local vanishing of forces in this and at subsequent stages.

The next step of structure degradation is due to the next sequence of fracture (elements 3 and 4) and the decrease of the gap from δ_2 to $\delta_3 = 0.03\delta_1$ (Fig. 8d). Therefore, the best location for actuators to control the first mode are elements 1 and 2 (and the corresponding symmetrically located elements) and the next two elements 3 and 4 (and the corresponding symmetrically located elements). The sequence of maximally deflected (and loaded by forced displacements) points A, B, C has been marked in Fig. 8.

Analogously, the second mode can be analyzed. The adjoint structure shaped as the second mode (Fig. 9b, only one-half of the antisymmetrically shaped structure is shown) was loaded by displacement forced in the joint A. The corresponding sequence of fractures (elements 1 and 2) causing the first drastic decrease of the gap from the value δ_1 to $\delta_2 = 0.14\delta_1$ is marked in Fig. 9c.

The PCA procedure has been applied to the three-dimensional example of an antenna backup truss structure composed of 102 elements (Fig. 10). The truss is assumed to be made of aluminum with Young's modulus $E = 10^7$ lb/in.^{2} and weight density $\rho = 0.1$ lb/in.^{2}. The cross-sectional area of the elements is $F = 1$ in.^{2}. For this configuration the first two eigenvalues are very close: $\lambda_1 \cong \lambda_2 \cong 138.486$ (rad/s)^2. The corresponding modes of vibration are shown in Figs. 11a and 12a, respectively. Only the upper layer of the truss structure is shown in these figures. Taking into account the solution of the PCA numerical procedure and the triple symmetry of the structure, the sequences of optimal locations for actuators for the first and second modes are shown in Figs. 11b and 12b, respectively. Suppressing deflections of the first mode, ideal actuators (without constraints imposed on values of active distortions) located in elements marked 1 (Fig. 11b) are most effective. Then, the sequence of best locations for actuators are marked 2 and 3. Analogously, the optimal locations of actuators to control the second mode are also located on the upper layer of the structure. The corresponding sequence of best locations (from 1 to 5) is marked in Fig. 12b.

Conclusions

The PCA method combined with the VDM simulation technique can efficiently detect optimal locations for actuators controlling the chosen modes of vibration (or modes of shape disturbances in quasistatic shape control applications). The result strongly depends on constraints imposed on the admissible state of virtual distortions (e.g., limited value of locally generated distortions).

The applied modal analysis of decomposing vibration coincides with a control strategy that allows decoupling of the real-time control process applicable to each mode independently. This aspect of structural control as well as the method of optimal location for sensors (combined with the PCA method) to maximize the accuracy of structural identification will be discussed in separate papers.

The spillover problem due to control of only some limited number of first modes, as well as corresponding problems of uncertainty of structural identification, can cause practical problems in robustness of active control. However, in flexible structures, the influence of higher modes on the macrostructural dynamic behavior is small.

The practical realization of the procedure described herein depends on the type of applied devices. For example, piezoelectric wafers can be placed along brittle fracture lines (not only points), which is very important in the problem of shell control, leading naturally (through the PCA procedure) to generation of lines of crack propagation. Moreover, some additional constraints should be imposed on the field of virtual distortions to simulate the real behavior of actuators.

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